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Usage of non-routine problem solving strategies at first grade level

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Abstract

The aim of this study is to research how the first graders solve non-routine problems and which strategies they use. We also aimed to investigate whether there are relationships between students' scores on non-routine problem solving strategies; their mathematical achievements and gender.

For this aim, a test was prepared including 6 non-routine problems and administered to 170 first grade students. The findings of the study show that the strategy used most successfully by the first graders is *look for a pattern* and there is a significant relationship between students' scores and their mathematical achievements. Besides, insignificant relationship found between students' scores and their gender.

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1. Introduction

In recent reform documents, there has been a strong emphasis on the acquisition of mathematical problem solving and reasoning skills and attitudes, and the ability to apply these skills in real-life situations as one of the major objectives of mathematics education at the elementary school level (Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts & Ratinckx, 1999). Moreover, a vast body of studies about mathematical problem solving shows that non-routine problems are the kind of problems which are most appropriate for developing the skills and abilities mentioned above (Altun, Bintas, Yazgan, Arslan, 2004).

According to Hershkowitz (2001), if the students solve a routine problem, they are likely to alternate between recognizing and building-with previously acquired structures. If they solve a non-routine problem, they might be constructing a new (to them) phenomenon and reflecting on it, on its internal structure, and on its external relationship to things they know already. Although routine problems can be used to fulfill particular didactical functions of teaching students to apply a certain procedure or a definition correctly; only through the careful use of non routine problems can students develop their problem solving ability (Stanic and Kilpatrick, 1988). Because, non-routine problems do not have straightforward solutions that can be seen at first sight on the contrary to routine problems which require bare calculations. Non-routine problems require a different reasoning and the application of certain heuristic strategies such as organizing or classifying data in different ways, recognizing patterns etc.

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To have a more detailed opinion about what a non-routine problem is, we can think on the solution of the following problem: “A male bee is born from an unfertilized egg, a female bee from a fertilized one. So, in other words, a male bee only has a mother, while a female bee has a mother and a father. How many total ancestors does a male bee have going ten generations back?” This problem can be solved with the help of the diagram in Figure 1. In this diagram F means female and M means male.

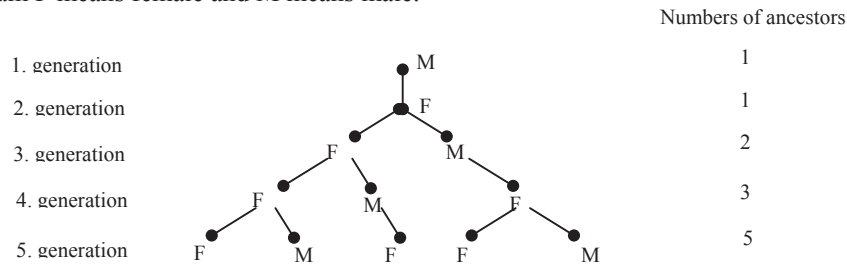


Figure 1.1 Solution of the non-routine problem sample

Without completing this diagram until the tenth generation, we can find out the solution by using the pattern among numbers of bees in every generation: Add the first and second number ($1 + 1 = 2$) to get the third number, add the second and third number to get the fourth number ($1 + 2 = 3$) and so on. According to this pattern, solution of the problem is 55.

In the literature, it is stated that solution methods which are used for non-routine problems can be classified and named based on their common properties. The most outstanding strategies among them are as follows: *Look for a pattern, make a systematic list, work backward, guess and check, draw a diagram, simplify the problem, make a table, eliminate the possibilities, reasoning, estimation* (Altun, Bintas, Yazgan, Arslan, 2004). In the solving process of some problems, only one strategy is used, while more strategies can be used for other problems. For example, *look for a pattern, draw a diagram, and simplify the problem* strategies are often used together, as it can be seen from the solution of problem about bees.

Many experimental studies have been carried out about the teaching of non-routine problem solving. For example, Higgins (1997) aimed in her study to investigate the effects of a year systematic instruction in problem solving on sixth and seventh grade students' attitudes and beliefs about problem solving and mathematics and on their problems solving ability. Comparisons were made between the students who engaged in this instruction and students who were taught mathematics in a more text-based, traditional manner. As a result, the students who had received the instruction displayed greater perseverance in solving problems, more positive attitudes about the usefulness of mathematics. Likewise, Verschaffel et al. developed and tested a learning environment for teaching and learning how to model and solve non-routine problems at 5th grade level. Pupils in the experimental group were taught a series of strategies, while pupils of a control group followed regular mathematics classes. The results indicated that the intervention had positive effects on different aspects of pupils' mathematical modeling and problem solving abilities. In another study by Follmer (2001) with 4th graders, it was aimed to investigate the impact that direct instruction in problem solving would have on enhancing students' mathematical thinking processes when solving non-routine problems. Findings showed that the instruction had a positive impact on students' use of cognitive strategies and their awareness of how they solved the problems.

To sum up, findings of experimental studies on non routine problem solving indicates that several strategies can be introduced at primary or middle school level, such as: *guess and check, draw a picture, act out the problem, use objects, choose an operation, solve a simpler problem, make a table, look for a pattern, make an organized list, write an equation, use logical reasoning, work backward* (Charles, Lester and O'Daffer, 1992).

In addition to experimental studies, there are some researches examining students' non-routine problem solving skills without any intervention. One of them was implemented by Cai (2003) with sixth grade students. In this study, the gender differences of U.S. and Chinese students in their solution processes of solving non-routine mathematical problems were examined. Results of the study showed that overall there were statistically significant gender differences (favoring males) on non-routine problem solving for the U.S. sample, but not for the Chinese sample. However, a more elaborate qualitative analysis of student responses to non-routine problems showed that male and

female students exhibit many similarities in their solution processes of solving non-routine problems. In a more recent study, Elia, van den Heuvel-Panhuizen and Kolovou (2009) sought strategy use, strategy flexibility and their relations with performance in non-routine problem solving at 4th grade level. Participants of the study were high-achieving students, and the authors proposed and investigated two types of strategy flexibility: inter-task flexibility (changing strategies across problems) and intra-task flexibility (changing strategies within problems). Findings of the study showed that students rarely applied strategies and they did not displayed the two types of flexibility to a large extent.

A general overview of the literature showed us there are many experimental researches about non-routine problem solving in which effects of an intervention were observed from different point of views and at different grade levels. However, we thought that there is need for research that aims to probe use of non-routine problem solving strategies by the first grade students without any instruction. Therefore, the aim of the present study is to investigate how first grade students solve non-routine problems, and which strategies they use. In addition, we aimed to investigate whether there are relationships among students' scores on non-routine problem solving strategies; their mathematical achievements and their gender.

2. Method

2.1 Participants

In the primary school in which our test was administered, there were 8 classes of first grades. Six of them were willing to participate in the study. In these classes, there were 183 students in total and test papers of 7 students were not evaluated due to their learning difficulties. The 176 remaining test papers were diminished to 170 to equalize the numbers of girls and boys. So, total of 170 students were examined in the present study.

2.2 Instrument

Before starting the study, a wide range of text books, books, web sites, articles etc. were thoroughly researched to determine strategies that are most prominent in literature. Based on results of this investigation, only four non-routine problem solving strategies were chosen: *look for a pattern*, *make a systematic list*, *work backward*, and *draw a diagram*. Moreover, we made a repertoire of problems by readjusting the problems used in previous studies.

We selected 8 problems from the repertoire, and carried out a pilot study with 40 students from a different primary school. After the pilot study, six problems were established to be included in the instrument. Four out of them required sole use of each of chosen strategies. One problem required common use of *look for a pattern* and *draw a diagram* strategies. Additionally, one of the problems was a non-routine division problem.

2.3 Procedure

The test was conducted by the first author in the middle of the school year, during regular school hours. In a quiet atmosphere, students were given 80 minutes to finish all problems because it was thought that they would need more time than one lesson since they had just learnt reading and writing during procedure. In addition, researcher made some explanations when needed to help children understand questions.

During the implementation of the test, almost none of the class teachers stayed in the classroom. Only one of them observed the procedure, but she did not give any assistance to students.

2.4 Data Analysis and Scoring

Based on solution processes that could be understood from the writings of students on test sheets, every question in the test except one question was scored as 0 (wrong or no answer), 1 (partly correct answer) and 2 (correct answer). *Look for a pattern* question was scored differently, because students were asked to write the next numbers in a number sequence according to pattern among given numbers. Therefore, we decided to code student solutions as 0 (wrong or no answer) and 1 (correct answer).

Since one of our aims is to investigate whether there is any link between non-routine problem solving achievement and mathematics achievement, we used students' mathematics scores at school and coded these scores as 1 (low), 2 (medium) and 3 (high). According to this coding, numbers of low, medium and high-achievers were 26, 62 and 82, respectively.

As for the analyses of data, first frequencies and percentages of scores were computed for each question. Thus, achievement levels for each strategy were determined. Additionally, cross-tabulation and χ^2 test were used to analyze relationships between non-routine problem solving - mathematics achievement, and non-routine problem solving - gender.

3. Findings

3.1 Strategy use

As mentioned in the previous section, we firstly calculated frequencies and percentages pertaining to each question to determine at which level first grade students can use non-routine problem solving strategies (Table 3.1). In this way, use of every strategy could be observed without any discrimination such as gender, or success at school.

Table 3. 1 Frequencies and percentages of scores for each question

Questions	Scores					
	0		1		2	
	f	%	f	%	f	%
Look for a pattern*	21	12.4	149	87.6	-	-
Make a systematic list	85	50	82	48.2	3	1.8
Non-routine division	100	58.8	28	16.5	42	24.7
Draw a diagram	145	85.3	11	6.5	14	8.2
Work backward	92	54.1	62	36.5	16	9.4
Look for a pattern- draw a diagram	135	79.4	14	8.2	21	12.4

*As explained in the Method section, this question was scored differently.

If we take into consideration frequencies and percentages of scores regarding exactly right answers, non-routine problem that was solved by the first graders most successfully is *look for a pattern* (%87.6) question. *Non-routine division* problem follows this problem in the sense of success level. However, big gap between success levels of these problems is conspicuous.

As for the other questions, it can be said that success levels of *draw a diagram*, *work backward*, and *draw a diagram-look for a pattern* questions are approximate to each other. Also, students failed in reaching exact answer of *make a systematic list* question.

3.2 Strategy use and mathematical achievement

In Table 3.2, frequencies and percentages that were computed separately for each achievement level are represented.

If data shown in Table II are evaluated together with results of χ^2 tests fulfilled for each question, findings can be summarized as follows:

-Values about *look for a pattern* question show that numbers of students who gave wrong answer are almost equal at each achievement level. Moreover, a statistically significant relationship was found between scores of this question and students' achievement level ($\chi^2_p > \chi^2_{(df;\alpha)}$; (9.658 > 5.99); df = 2 and $\alpha = .05$).

- When we have a general view of scores on *make a systematic list* question, we can say that success levels are very low at each achievement level, since only 3 high-achievers could completely solve this problem. Additionally, there is not any significant relationship between scores and students achievement level ($\chi^2_p < \chi^2_{(df;\alpha)}$; (8.84 < 9.49); df = 4 and $\alpha = .05$).

-As for *non-routine division* problem, interesting point is that numbers of wrong answers are quite high at each achievement level. Besides, there is a significant relationship between success level at this question and achievement level. ($\chi^2_p > \chi^2_{(df;\alpha)}$ (20.211 > 9.49); df = 4 and $\alpha = .05$)

Table 3.2 Frequencies and percentages based on students' mathematical achievement

Questions	Scores	Achievement levels						Total (f)
		1 (low)		2 (medium)		3 (high)		
		f	%	F	%	f	%	
Look for a pattern	0	8	38.1	6	28.6	7	33.3	21
	1	18	12.1	56	37.6	75	50.3	149
Make a systematic list	0	17	20	35	41.2	33	38.8	85
	1	9	11	27	33	46	56	82
	2	0	0	0	0	3	100	3
Non-routine division	0	21	21	43	43	36	36	100
	1	3	10.7	11	39.3	14	50	28
	2	2	4.8	8	19	32	76.2	42
Draw a diagram	0	26	17.9	57	39.3	62	42.8	145
	1	0	0	2	18.2	9	81.8	11
	2	0	0	3	21.4	11	78.6	14
Work backward	0	15	16.3	46	50	31	33.7	92
	1	11	17.7	14	22.6	37	59.7	62
	2	0	0	2	12.5	14	87.5	16
Look for a pattern- draw a diagram	0	25	18.5	52	38.5	58	43	135
	1	1	7.1	4	26.6	9	64.3	14
	2	0	0	4	28.6	15	71.4	19

- From the frequencies and percentages regarding *draw a diagram* question, it can be understood that low and medium achievers had a serious setback at this question. χ^2 test results implicates a strong link between scores and achievement level ($\chi^2_p > \chi^2_{(df;\alpha)}$ (12.825 > 9.49); df = 4 and $\alpha = .05$).

- Scores which students got from *work backward* and *look for a pattern - draw a diagram* questions imply two points: Low and medium achievers' performances are very low at these questions and there are significant relationships between scores of each question and achievement level (For *work backward* $\chi^2_p > \chi^2_{(df;\alpha)}$ (23.998 > 9.49), df = 4 and $\alpha = .05$; for *look for a pattern - draw a diagram* $\chi^2_p > \chi^2_{(df;\alpha)}$ (9.559 > 9.49); df = 4 and $\alpha = .05$).

3.3 Strategy use and gender

In Table 3.3, frequencies and percentages that were computed separately for each gender are represented.

Table 3.3 Frequencies and percentages based on students' gender

Questions	Gender											
	Female						Male					
	0		1		2		0		1		2	
	f	%	f	%	F	%	f	%	f	%	f	%
Look for a pattern	11	13	74	87	-	-	10	11.8	75	88.2	-	-
Make a systematic list	38	44.7	45	52.9	2	2.4	47	55.3	37	43.5	1	1.2
Non-routine division	43	50.6	18	21.2	24	28.2	57	67	10	11.8	18	21.2
Draw a diagram	71	83.5	6	7.1	8	9.4	74	87	5	5.9	6	7.1
Work backward	43	50.6	30	35.3	12	14.1	49	57.6	32	37.6	4	4.8
Look for a pattern- draw a diagram	63	74.1	8	9.4	14	16.5	72	84.7	6	7.1	7	8.2

From Table 3.3, it can be seen that most successfully solved problems by males and females were same as the general range: *Look for a pattern* and *non-routine division*. Besides, a crucial lowness can be observed at each gender's scores of *make a systematic list* question. Speaking generally, scores of male and females at each question are close to each other. Thus, χ^2 values of each question in Table 3.4 imply insignificant relationship between strategy use and gender at .95 significance level ($\chi^2_p < \chi^2_{(df;\alpha)}$).

Table 3.4 χ^2 test results about relationship between strategy use and gender

Questions	χ^2_p	df	$\chi^2 (df; \alpha)$
Look for a pattern	.054	1	3.85
Make a systematic list	2.130	2	5.99
Non-routine division	5.103	2	5.99
Draw a diagram	.439	2	5.99
Work backward	4.456	2	5.99
Look for a pattern- draw a diagram	3.219	2	5.99

4. Conclusions

In the present study, we aimed to obtain in-depth information about first grade students' usage of non-routine problem solving strategies. In this context, we also investigated whether there are relationships between students' scores on non-routine problem solving strategies; their mathematical achievements and their gender. To these aims, firstly success levels for every strategy were generally assessed through frequencies and percentages. Based on these assessments, one of the most important finding of our study is that first grade students could not tackle with non-routine problems successfully without any intervention. This finding is coherent to finding of study carried out by Elia et al. (2009) at a different grade level. The marginal place of non-routine problem in the Turkish mathematics text books and curriculum could offer an explanation for this result.

There is only one strategy that pupils use effectively: *look for a pattern*. When exactly right answers are considered, usage levels of all other strategies are below 25%, while that of *look for a pattern strategy* is 87.6%. This success can be attributed to place allocated for this strategy in our primary school math curriculum and textbooks.

Although they constituted almost half of the participants, even high achievers did not exhibit high performances on non-routine problem solving as expected. Elia et al. (2009) also studied with high-achievers and they reached the similar result that students infrequently executed the strategies. On the other hand, performances of high-achieving students were higher than that of students at other achievement levels. Consequently, significant relationships were found between usage levels of strategies and achievement levels. There was only one exception: Usage level of *make a systematic list* strategy did not change significantly depending on achievement level.

As for finding about relationship between gender and success in non-routine problem solving, we found that there is not any interaction between these two variables. Cai (2003) also found that gender has not any impact on Chinese students' processes of non-routine problem solving.

Our research has some limitations as well. For example, we used only six questions in the test since first graders has a limited period of concentration. We could have got more detailed information about strategy use of these students, if we had asked more questions. Moreover, same students could have been observed within specific time intervals. These can be realized in future research. Another deficiency of our study is that numbers of low, medium, and high-achievers are not equal. If they were equal, we may have had more sound knowledge about relationship between non-routine problem success and mathematics achievement at school.

Based on the results of this study, we can summarize our recommendations as follows:

1. Non-routine problem solving strategies should be taught from the beginning of the first grade.
2. Teachers should have more information about non-routine problem solving strategies so that they can teach these strategies to their students.
3. Non-routine problems and strategies about their solutions should have more importance in math textbooks and primary school math curriculum.

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